【10920 趙啟超教授離散數學 / 第 1 堂版書與簡報】

EECS 2060 Discrete Mathematics

Course Information

趙啟超
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National Tsing Hua University

People, Time, & Place

- **Instructor:**
  趙啟超, Chi-chao Chao, *ccc@ee.nthu.edu.tw*, x31158, Delta 863.

- **Lecture Hours:**
  Wednesday, 10:10am to 12:00pm; Friday, 11:10am to 12:00pm; Delta 215.

- **Instructor's Office Hours:**
  To be announced.

People, Time, & Place *(cont.)*

- **Teaching Assistants:**
  To be announced.

- **TAs' Office Hours:**
  To be announced.

Prerequisite

- High-School Mathematics
- Calculus (preferred)

Course Contents

This course gives an introduction to the essentials of discrete and combinatorial mathematics.

- *Fundamentals* (13 hours)
  - Logic
  - Set theory
  - Mathematical induction
  - Functions: definitions, pigeonhole principle
  - Relations: definitions and properties, equivalence relations, partial orders

Course Contents *(cont.)*

- ***Enumeration*** (15 hours)
  - **Principles of counting**
  - **Principle of inclusion and exclusion**
  - **Recurrence relations:** homogeneous recurrence relations, nonhomogeneous recurrence relations
  - **Generating functions:** generating functions for solving recurrence relations, generating functions for enumeration, partitions of integers
  - **Complexity of algorithms**

Course Contents *(cont.)*

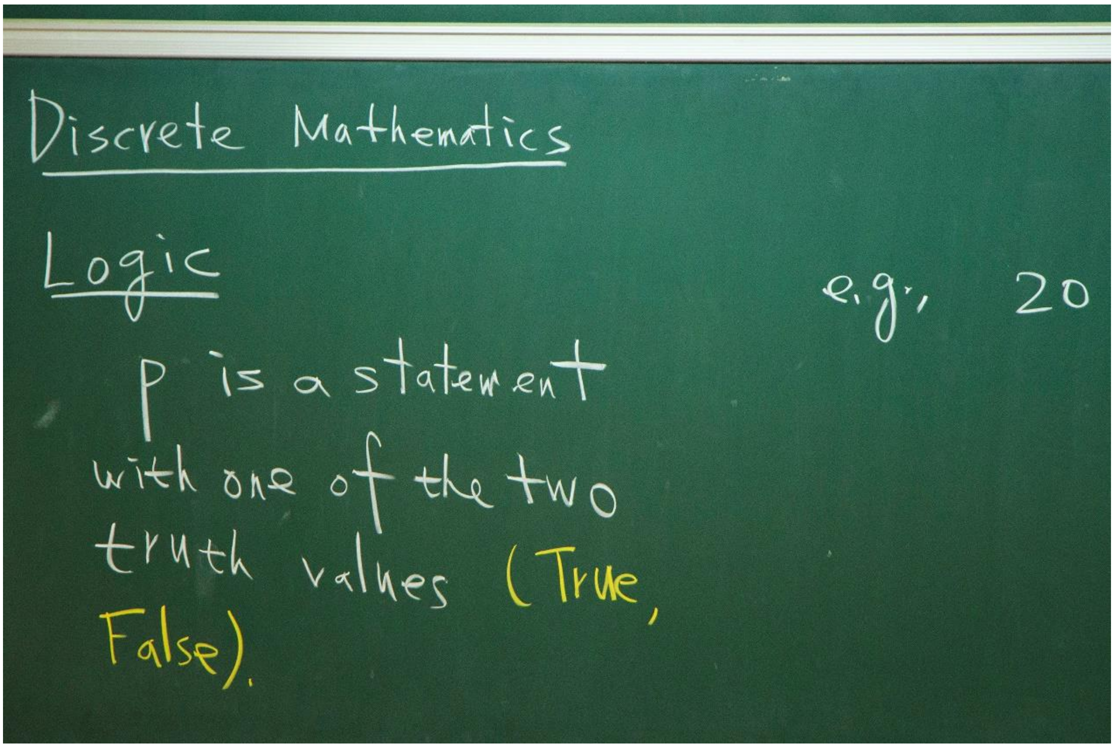
- ***Graph theory*** (22 hours)
  - **Introduction:** definitions and properties, graph isomorphism, Euler trails and circuits, planar graphs
  - **Trees:** definitions and properties, rooted trees, spanning trees, trees and sorting
  - **Optimization and matching:** shortest-path problem, minimal spanning trees, matching problem, maximum flow problem

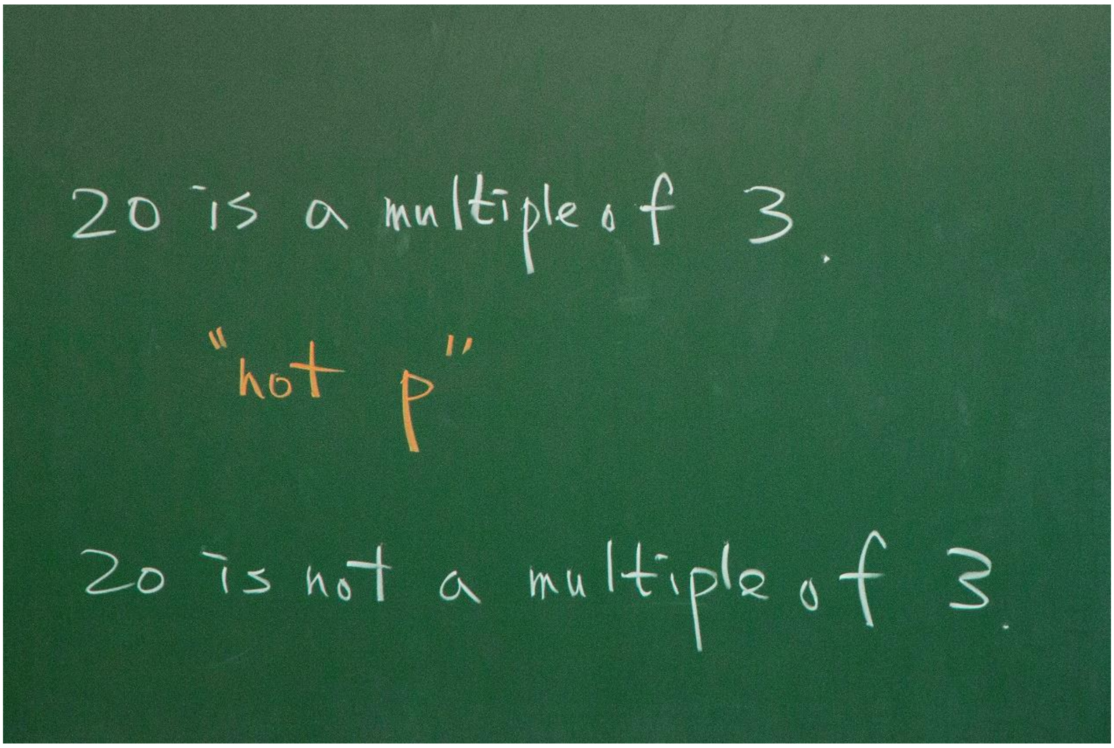
Textbook & References

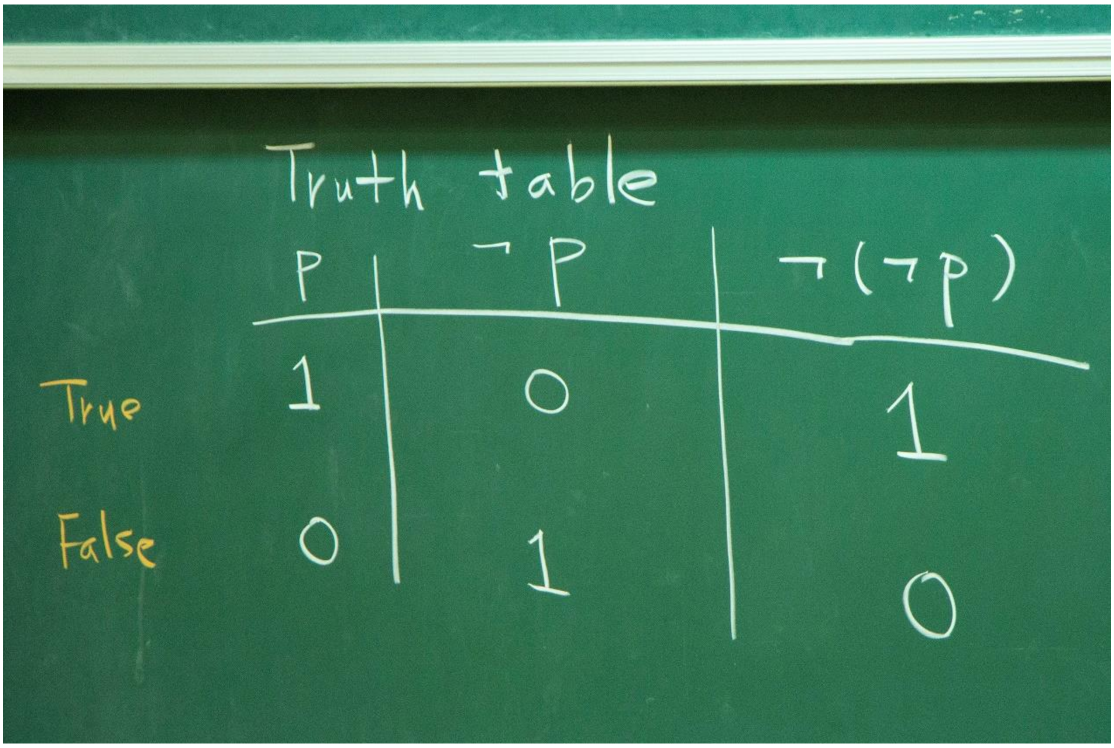
R. P. Grimaldi, *Discrete and Combinatorial Mathematics: An Applied Introduction, 5th ed*. Boston: Pearson Addison Wesley, 2004.

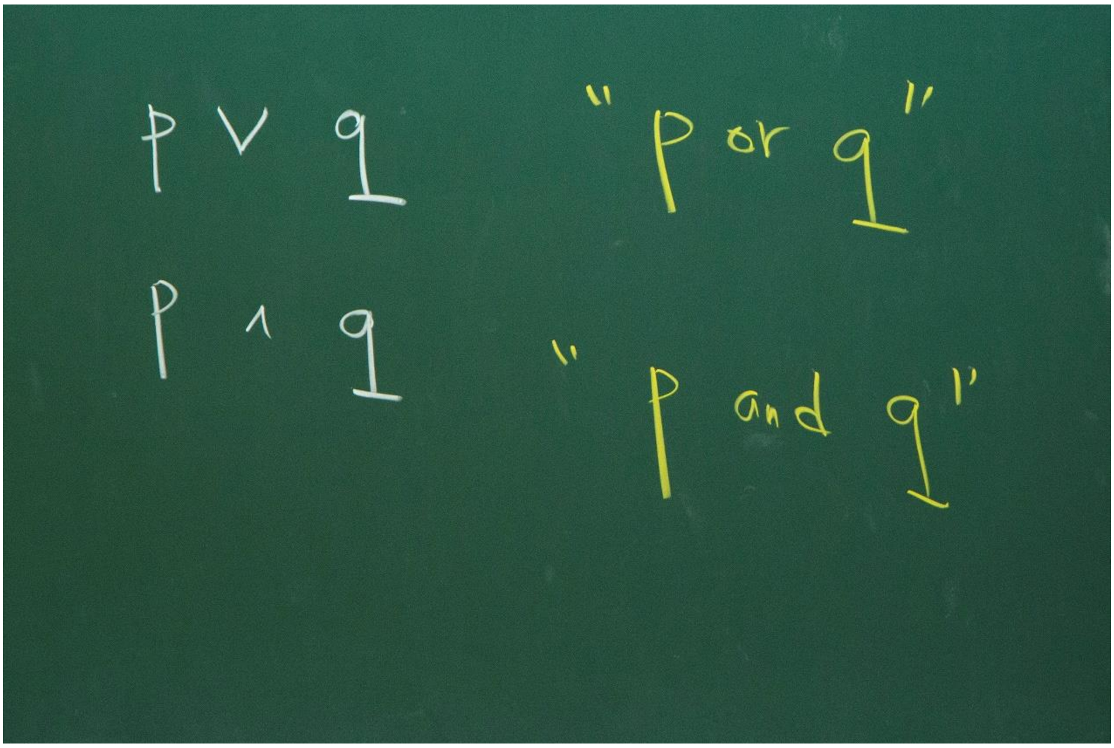
References:

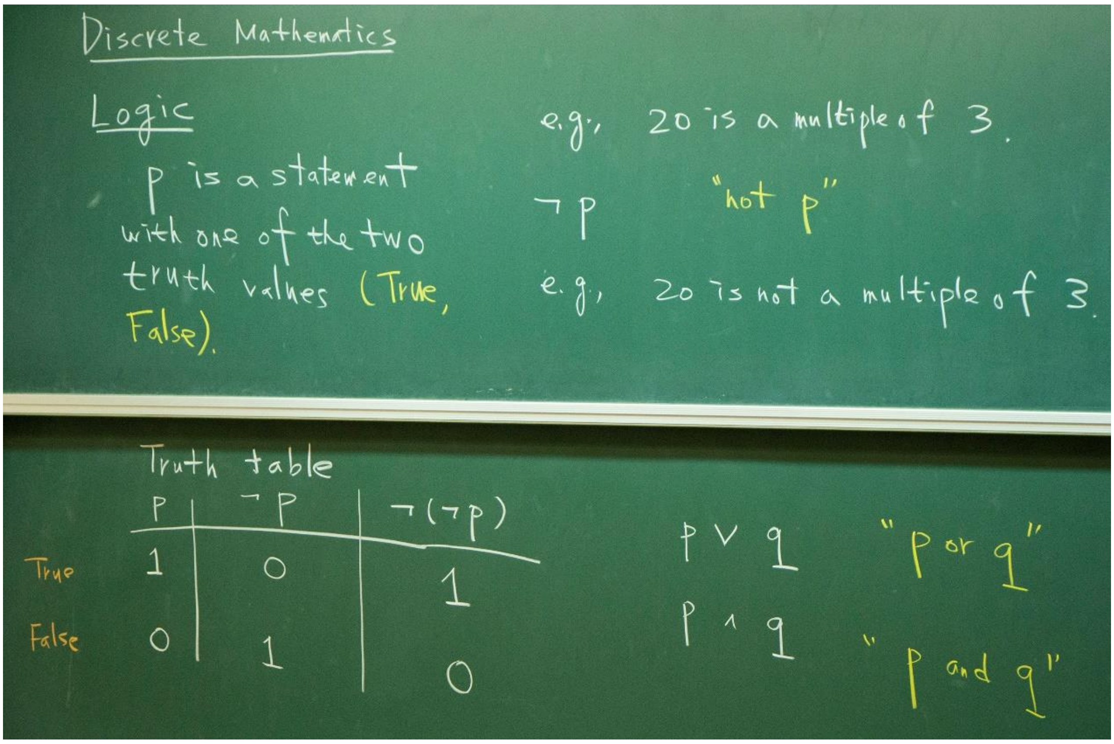
- R. J. McEliece, R. B. Ash, and C. Ash, *Introduction to Discrete Mathematics*. New York: Random House, 1989.
- N. L. Biggs, *Discrete Mathematics, 2nd ed*. New York: Oxford University Press, 2002.
- C. L. Liu, *Elements of Discrete Mathematics, 2nd ed*. New York: McGraw-Hill, 1985.
- C. L. Liu, *Introduction to Combinatorial Mathematics*. New York: McGraw-Hill, 1968.
- K. H. Rosen, *Discrete Mathematics and Its Applications, 8th ed*. New York: McGraw-Hill, 2019.
- R. L. Graham, D. E. Knuth, and O. Patashnik, *Concrete Mathematics: A Foundation for Computer Science, 2nd ed*. Reading, MA: Addison-Wesley, 1994.

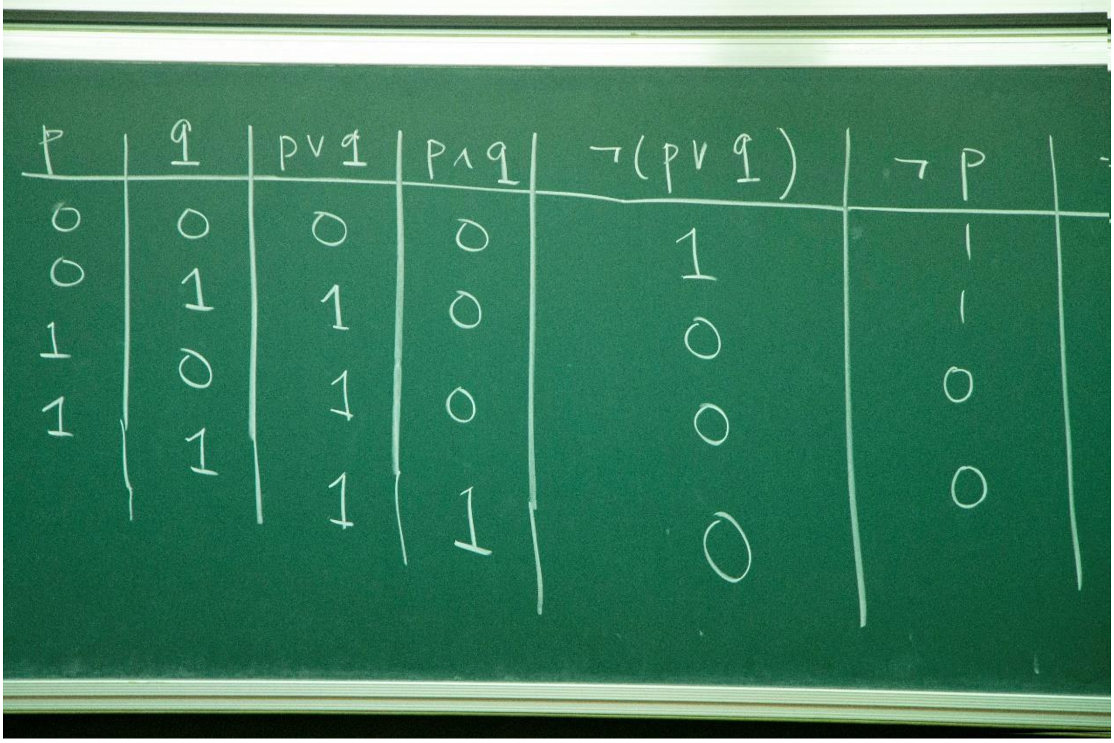


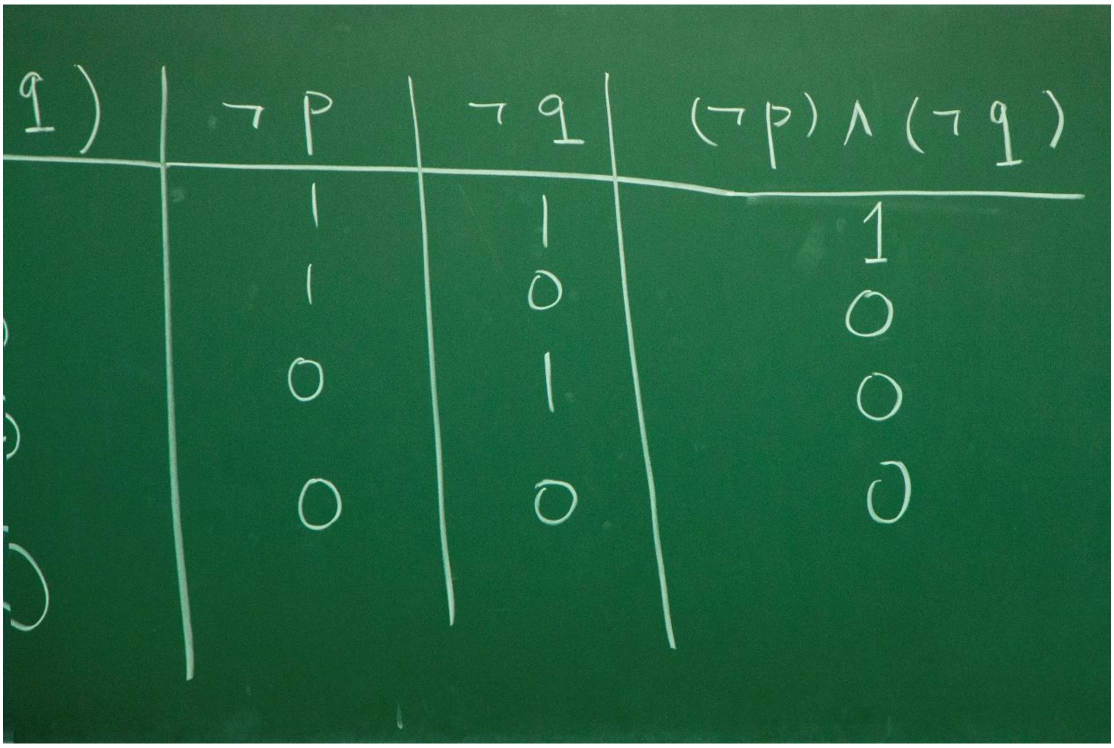

20 is a multiple of 3.
"not p"
20 is not a multiple of 3.


Truth table
p
¬p
¬(¬p)
True
1
0
1
False
0
1
0


p ∨ q
"p or q"
p ∧ q
"p and q"


Discrete Mathematics
Logic
p is a statement with one of the two truth values (True, False).
e.g., 20 is a multiple of 3.
¬p "not p"
e.g., 20 is not a multiple of 3.
Truth table
p ¬p ¬(¬p)
True 1 0 1
False 0 1 0
p ∨ q "p or q"
p ∧ q "p and q"


p | q | p∨q | p∧q | ¬(p∨q) | ¬p
0 | 0 | 0 | 0 | 1 | 1
0 | 1 | 1 | 0 | 0 | 1
1 | 0 | 1 | 0 | 0 | 0
1 | 1 | 1 | 1 | 0 | 0


¬p | ¬q | (¬p)∧(¬q)
1 | 1 | 1
1 | 0 | 0
0 | 1 | 0
0 | 0 | 0

Similarly, $\neg(p \wedge q)$ and $(\neg p) \vee (\neg q)$ are logically equivalent.

$$\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$$

n's law

We say that $\neg(p \vee q)$ and $(\neg p) \wedge (\neg q)$ are logically equivalent.

$$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$$

DeMorgan's law

Similarly

$(\neg p) \vee ($

equivalen

$\neg(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$
0	0	0	0	1	1	1	1
0	1	1	0	0	1	0	0
1	0	1	0	0	0	1	0
1	1	1	1	0	0	0	0

We say that $\neg(p \vee q)$ and $(\neg p) \wedge (\neg q)$ are logically equivalent.

Similarly, $\neg(p \wedge q)$ and $(\neg p) \vee (\neg q)$ are logically equivalent.

$$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q) \qquad \neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$$

DeMorgan's law

$p \Rightarrow q$ "if p then q" "p implies q"

p is called the premise and q the conclusion.

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

0	1	1
1	0	0
1	1	1

If the premise is false, then the implication $p \Rightarrow q$ is true regardless of whether q is true or not.

"innocent until proven guilty"

$p \Rightarrow q$ "if p then q" "p implies q"

p is called the premise and q the conclusion.

p	q	$p \Rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

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0	1	1
1	0	0
1	1	1

If the premise is false, then the implication $p \Rightarrow q$ is true regardless of whether q is true or not.

"innocent until proven guilty"

Example

(1) If elephants can fly, then the Dodgers will win the World Series this year.

(2) If eleph

the Dodge

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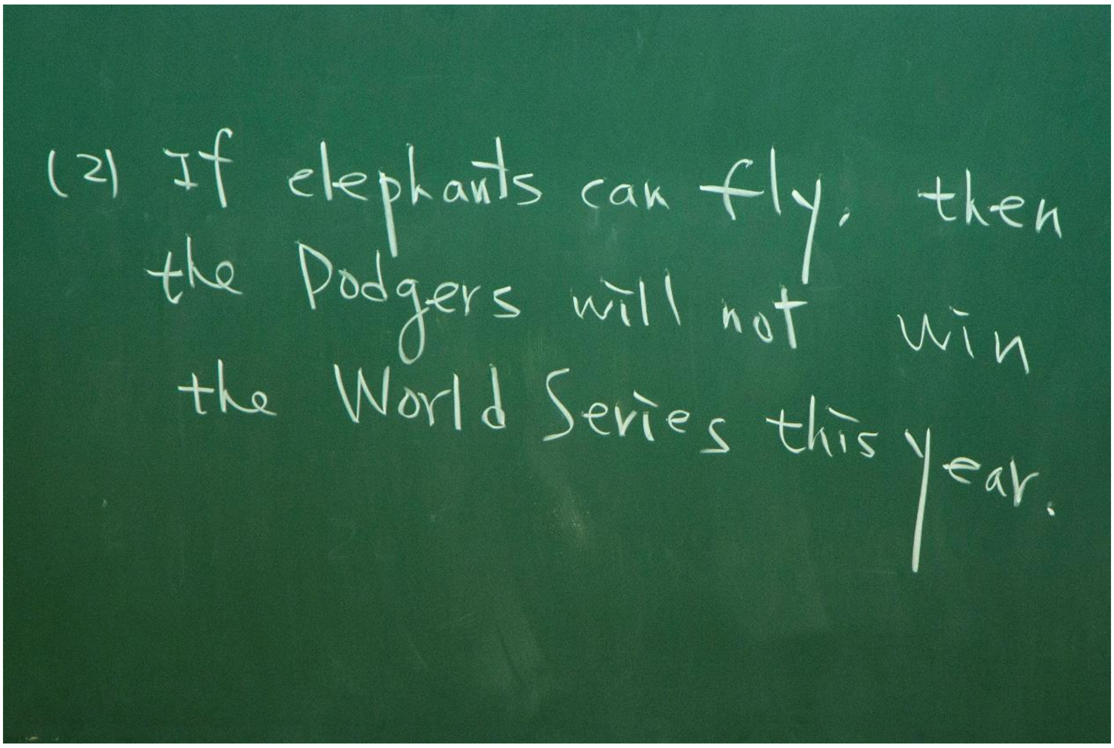
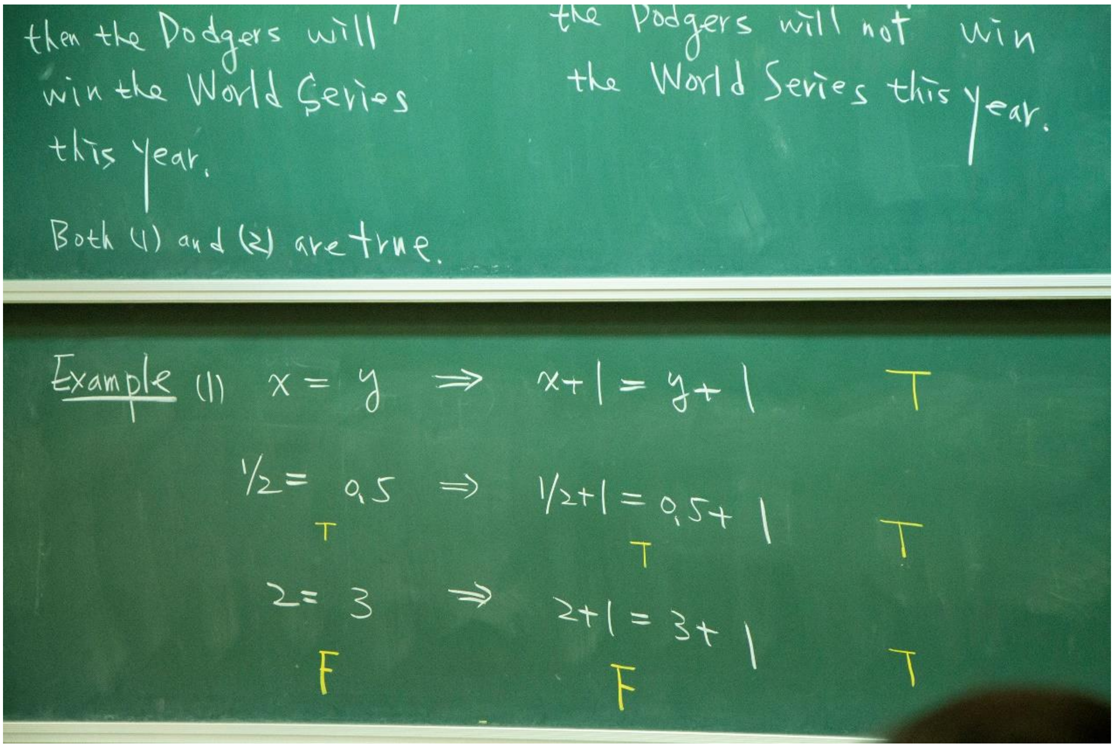
Both (1) and (2) are true.


(2) If elephants can fly, then
the Dodgers will not win
the World Series this year.


then the Dodgers will
win the World Series
this year.
Both (1) and (2) are true.
the Dodgers will not win
the World Series this year.
Example (1) $x = y \Rightarrow x+1 = y+1$ T
$1/2 = 0.5 \Rightarrow 1/2+1 = 0.5+1$ T
T T
$2 = 3 \Rightarrow 2+1 = 3+1$ T
F F

Example

(1) If elephants can fly, then the Dodgers will win the World Series this year.

(2) If elephants can fly, then the Dodgers will not win the World Series this year.

Both (1) and (2) are true.

Example (1)

$x = y$	$\Rightarrow$	$x+1 = y+1$		T
$1/2 = 0.5$	$\Rightarrow$	$1/2+1 = 0.5+1$		T
T		T		
$2 = 3$	$\Rightarrow$	$2+1 = 3+1$		T
F		F		

(2)

$x = y$	$\Rightarrow$	$xz = yz$		T
$2 = 3$	$\Rightarrow$	$2 \cdot 0 = 3 \cdot 0$		T
F		T		

$q \Rightarrow p$ "converse of $p \Rightarrow q$)

$p \Leftrightarrow q$ " p if and only if q"

($p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$)

$\neg q \Rightarrow \neg p$ " contrapositive of $p \Rightarrow q$"

(2) $x = y \Rightarrow xz = yz$ T

$\underset{F}{2 = 3} \Rightarrow \underset{T}{2 \cdot 0 = 3 \cdot 0}$ T

$q \Rightarrow p$ "converse of $p \Rightarrow q$)

$p \Leftrightarrow q$ " p if and only if q"

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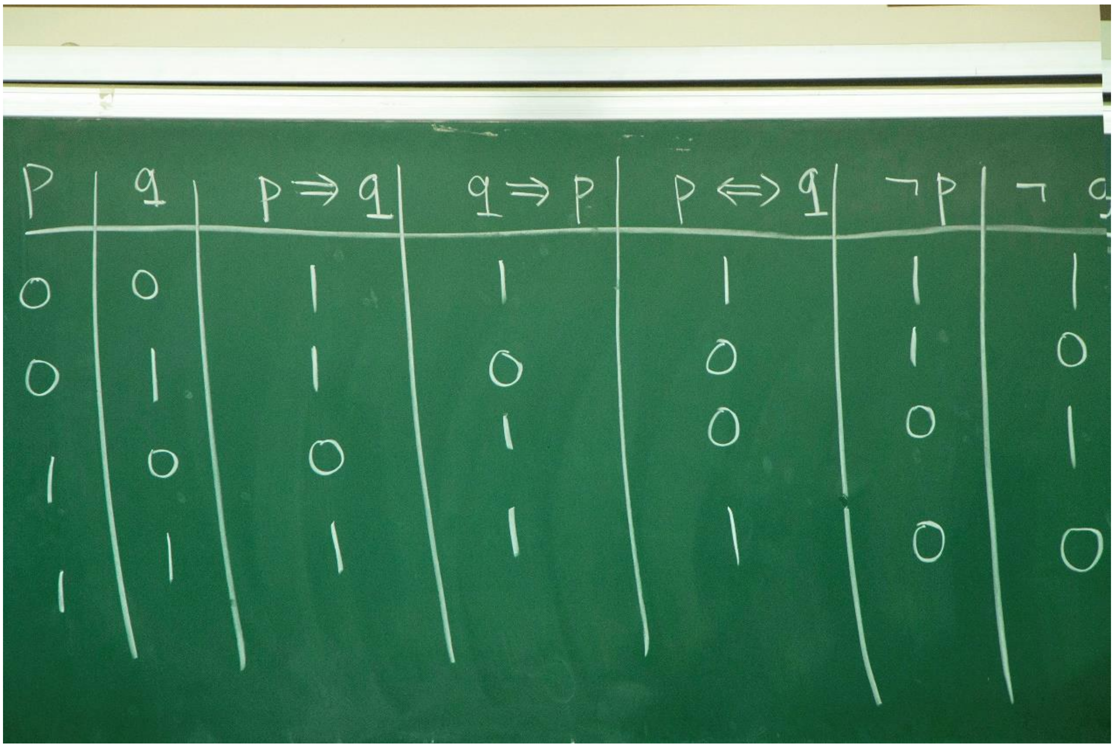
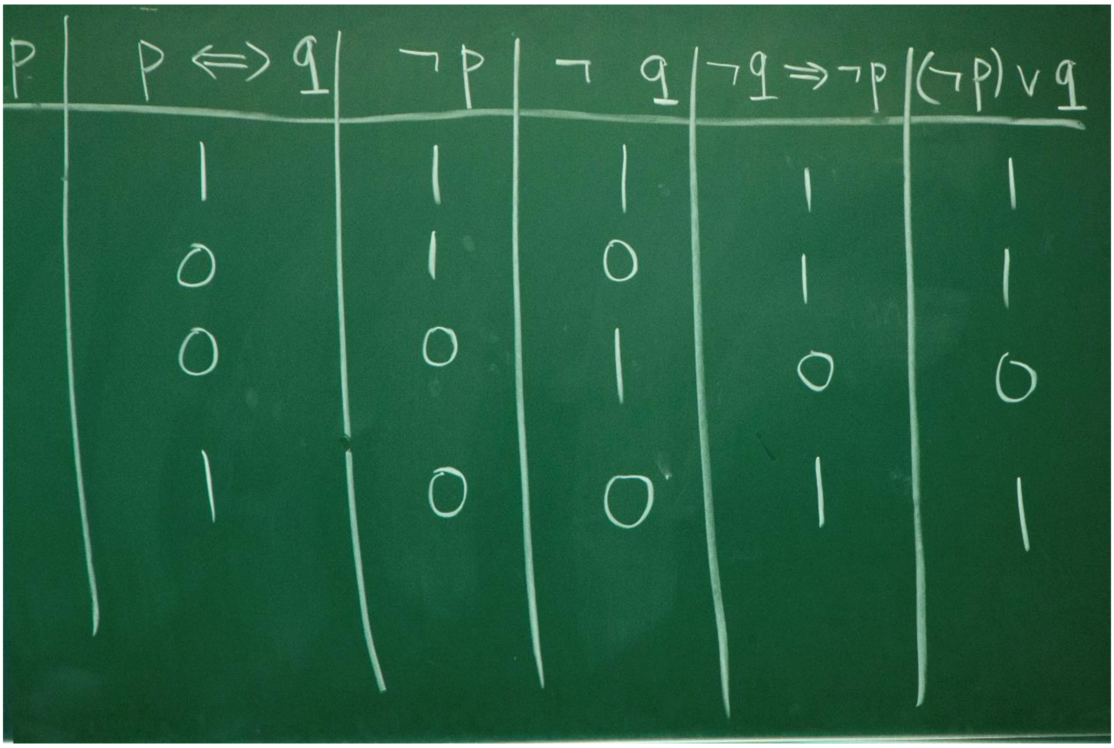
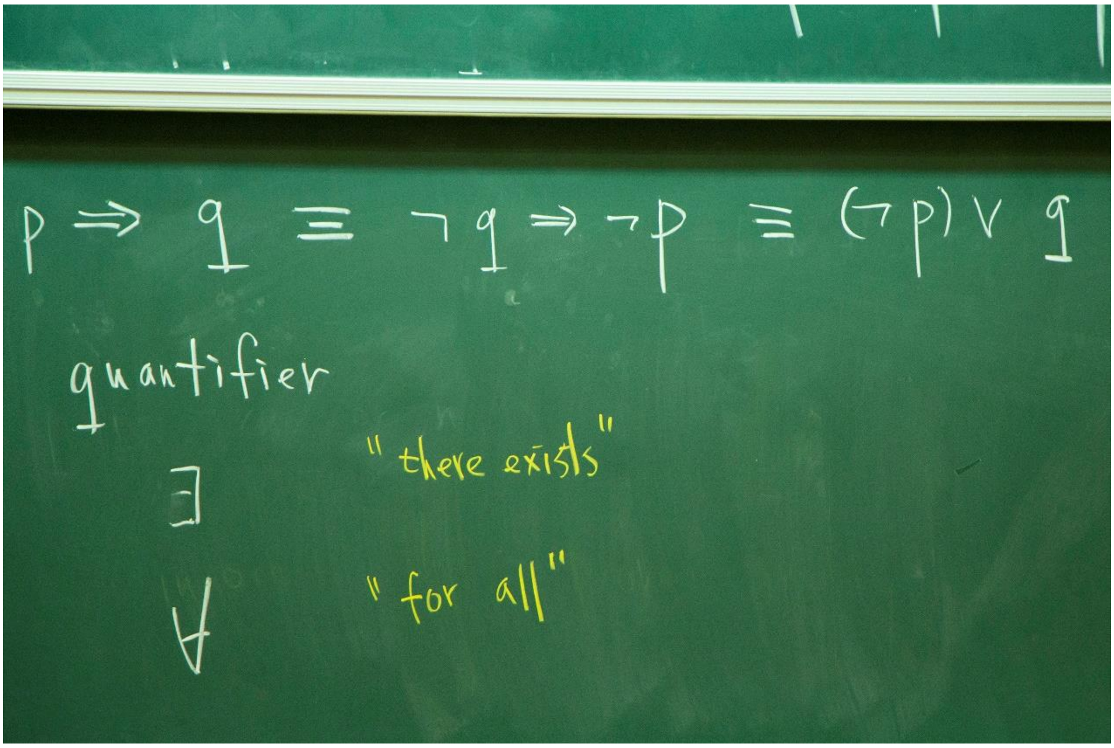
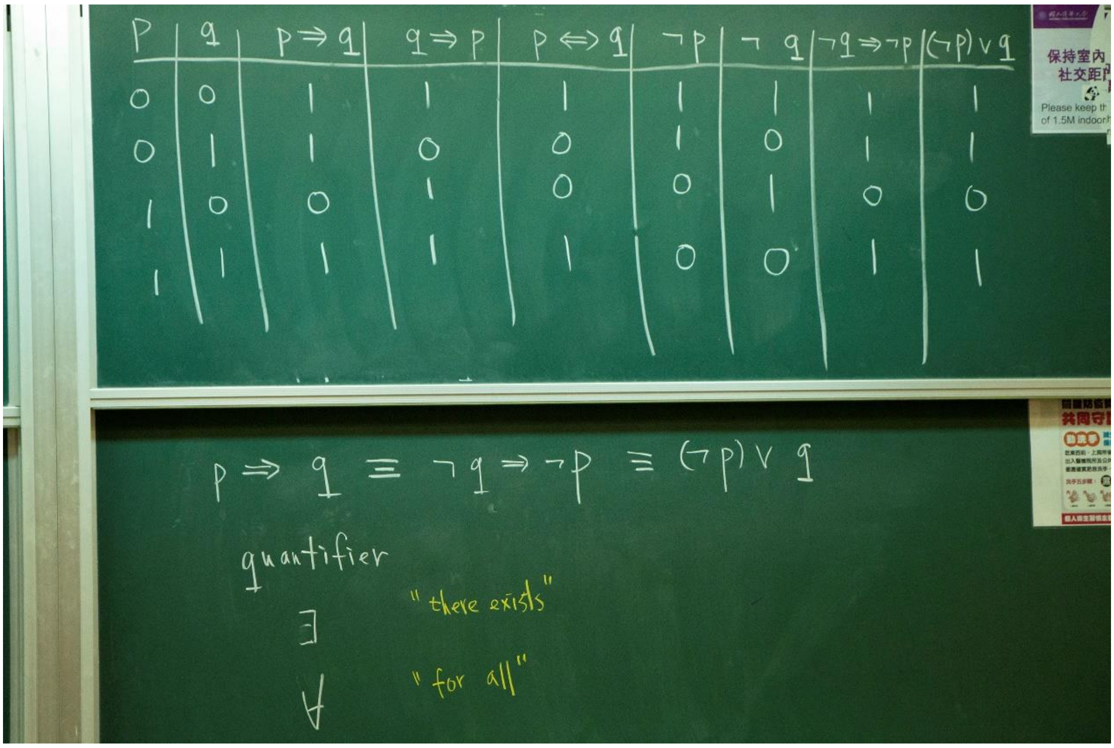


p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \Leftrightarrow q$	$\neg p$	$\neg q$
0	0	1	1	1	1	1
0	1	1	0	0	1	0
1	0	0	1	0	0	1
1	1	1	1	1	0	0



p	$p \Leftrightarrow q$	$\neg p$	$\neg q$	$\neg q \Rightarrow \neg p$	$(\neg p) \vee q$
	1	1	1	1	1
	0	1	0	1	1
	0	0	1	0	0
	1	0	0	1	1


p ⇒ q ≡ ¬q ⇒ ¬p ≡ (¬p) ∨ q
quantifier
∃ "there exists"
∀ "for all"


p | q | p⇒q | q⇒p | p⇔q | ¬p | ¬q | ¬q⇒¬p | (¬p)∨q
0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1
0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1
1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0
1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1
p ⇒ q ≡ ¬q ⇒ ¬p ≡ (¬p) ∨ q
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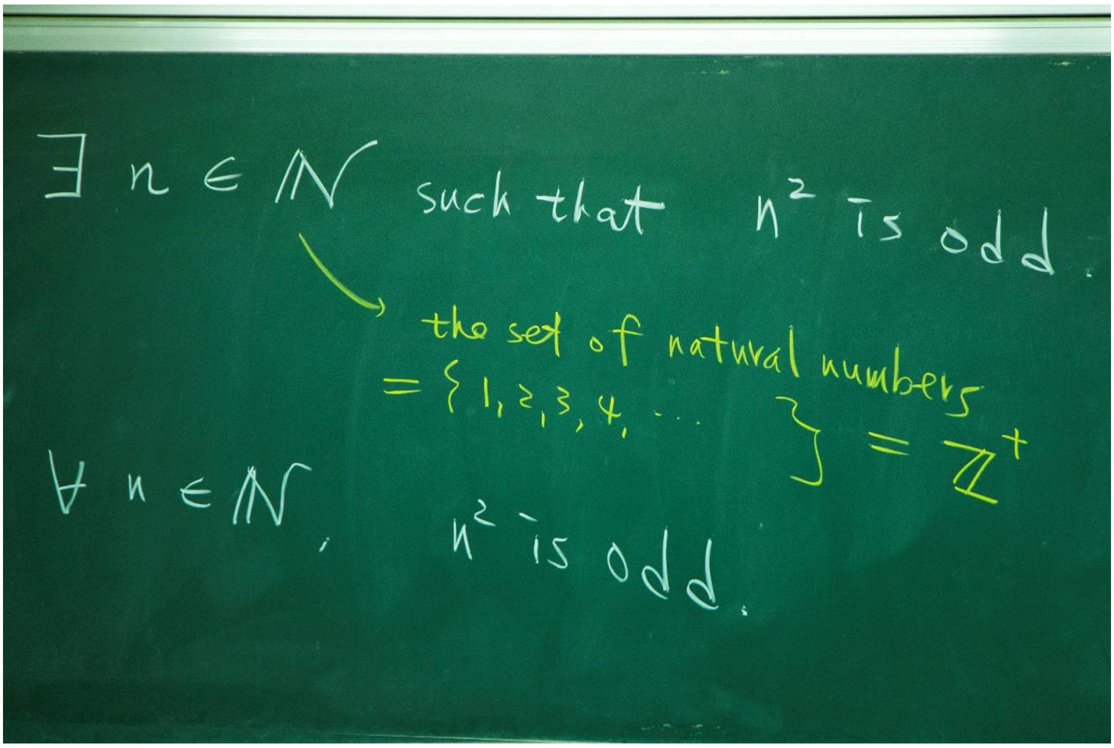
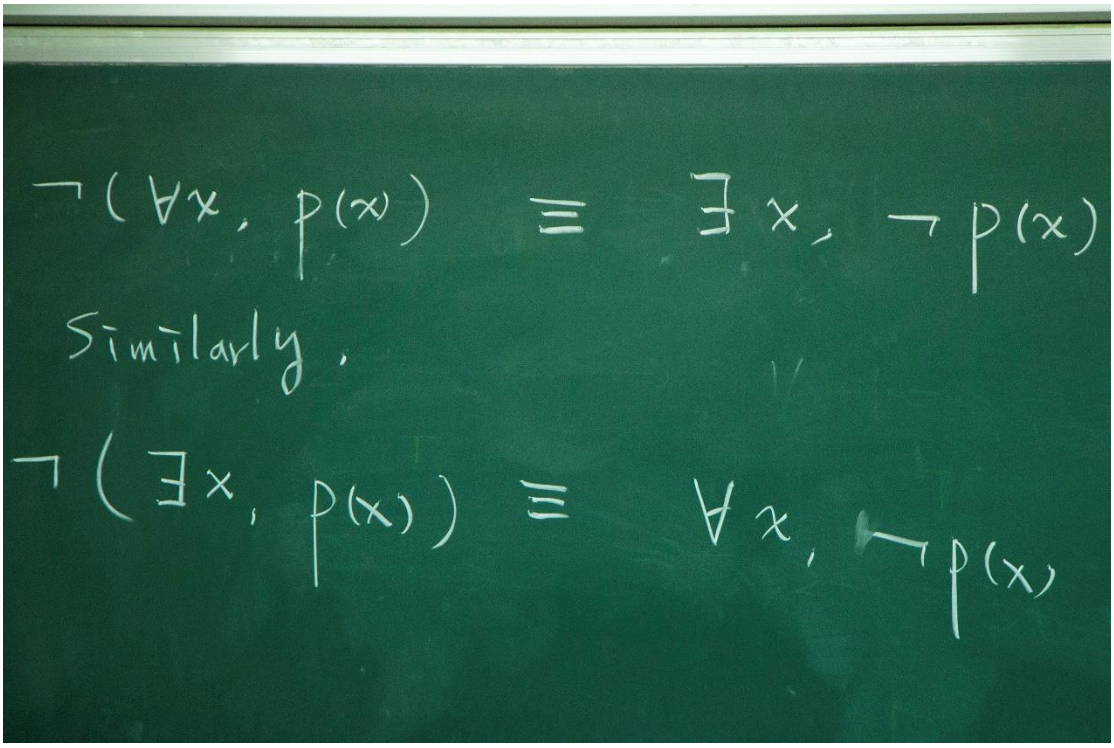
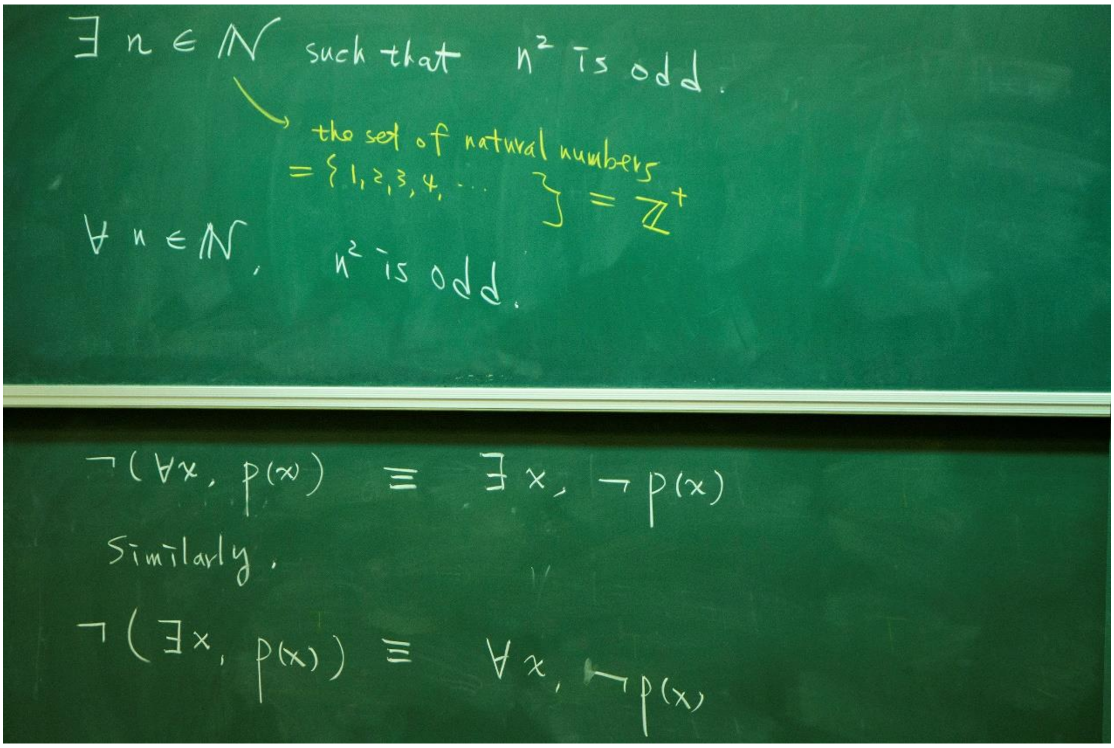

$\exists\, n \in \mathbb{N}$ such that n^2 is odd.
the set of natural numbers
$= \{1, 2, 3, 4, \ldots\} = \mathbb{Z}^+$
$\forall\, n \in \mathbb{N}$, n^2 is odd.


$\neg(\forall x,\ p(x)) \equiv \exists x,\ \neg p(x)$
Similarly,
$\neg(\exists x,\ p(x)) \equiv \forall x,\ \neg p(x)$


$\exists n \in \mathbb{N}$ such that n^2 is odd.
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Similarly,
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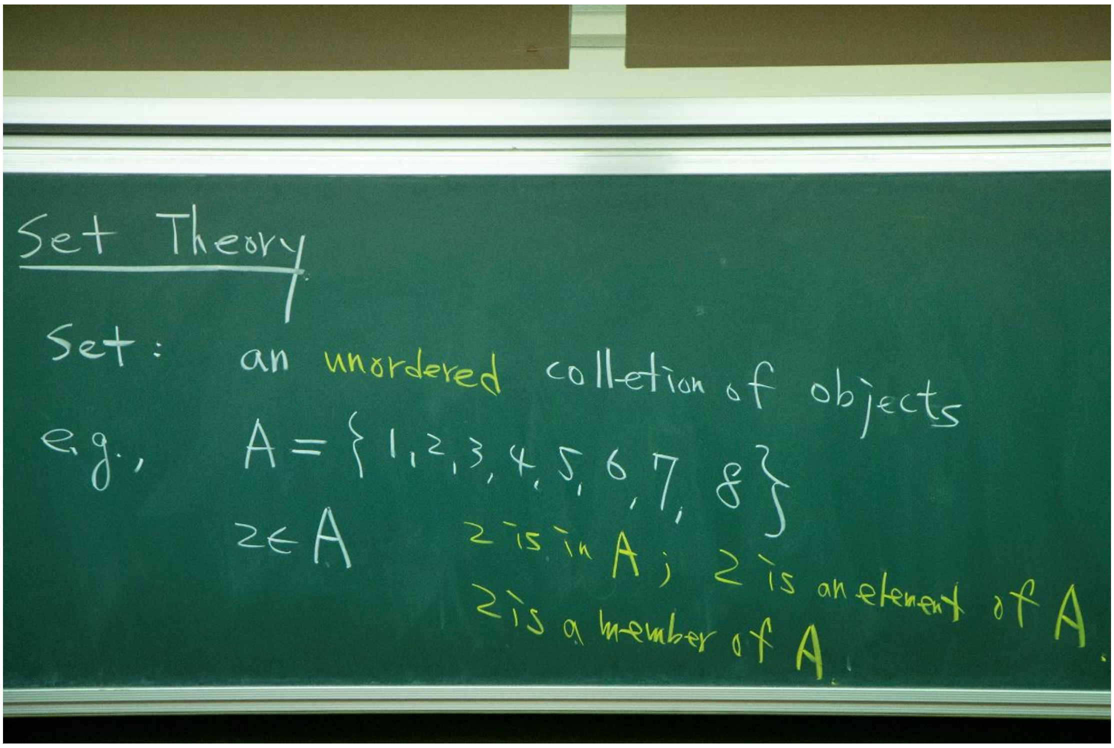
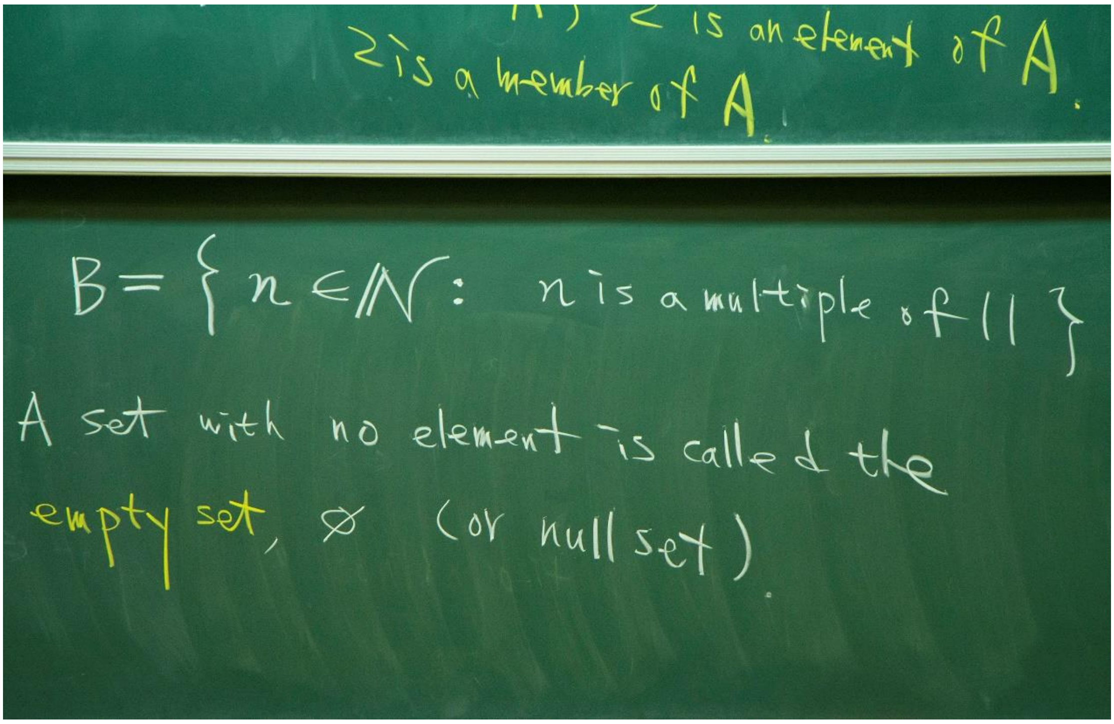
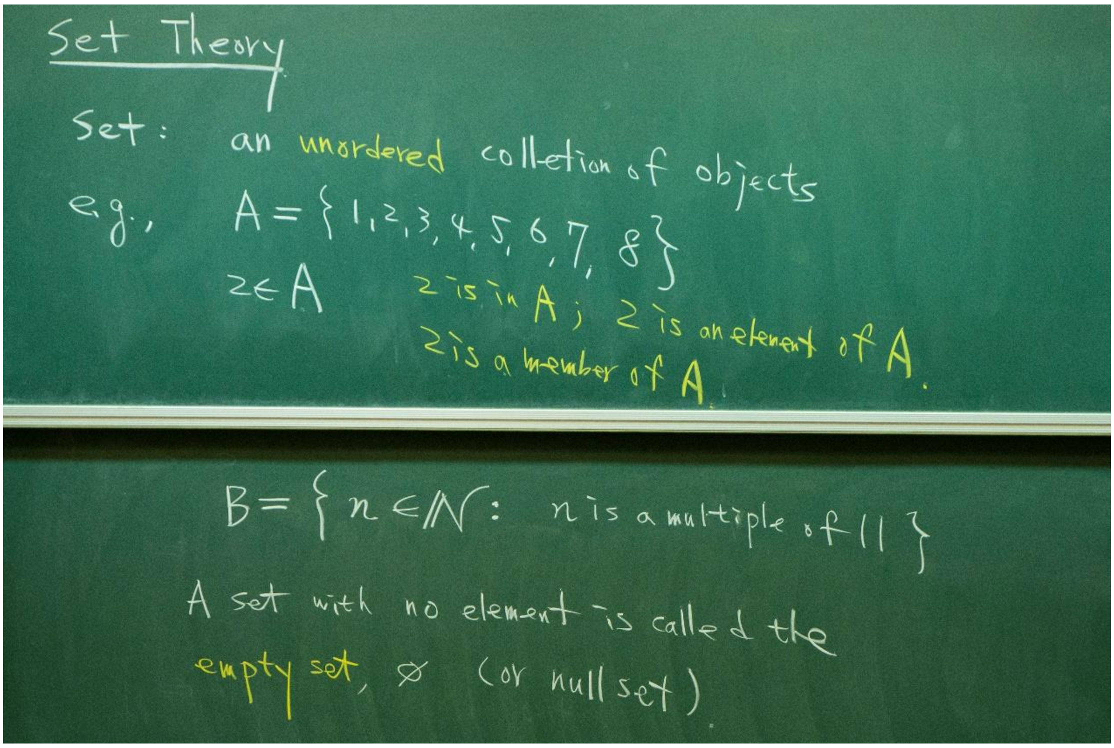

Set Theory
Set: an unordered colletion of objects
e.g., $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$2 \in A$ 2 is in A; 2 is an element of A.
2 is a member of A.


2 is an element of A.
2 is a member of A.
B = { n ∈ N : n is a multiple of 11 }
A set with no element is called the
empty set, ∅ (or null set).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Set Theory

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2 is a member of A.

$B = \{n \in \mathbb{N} : n \text{ is a multiple of } 11\}$

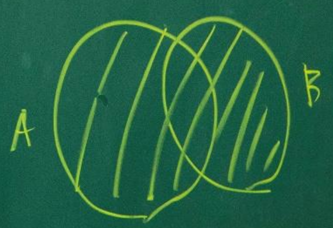
A set with no element is called the empty set, $\varnothing$ (or null set)

Set operations:

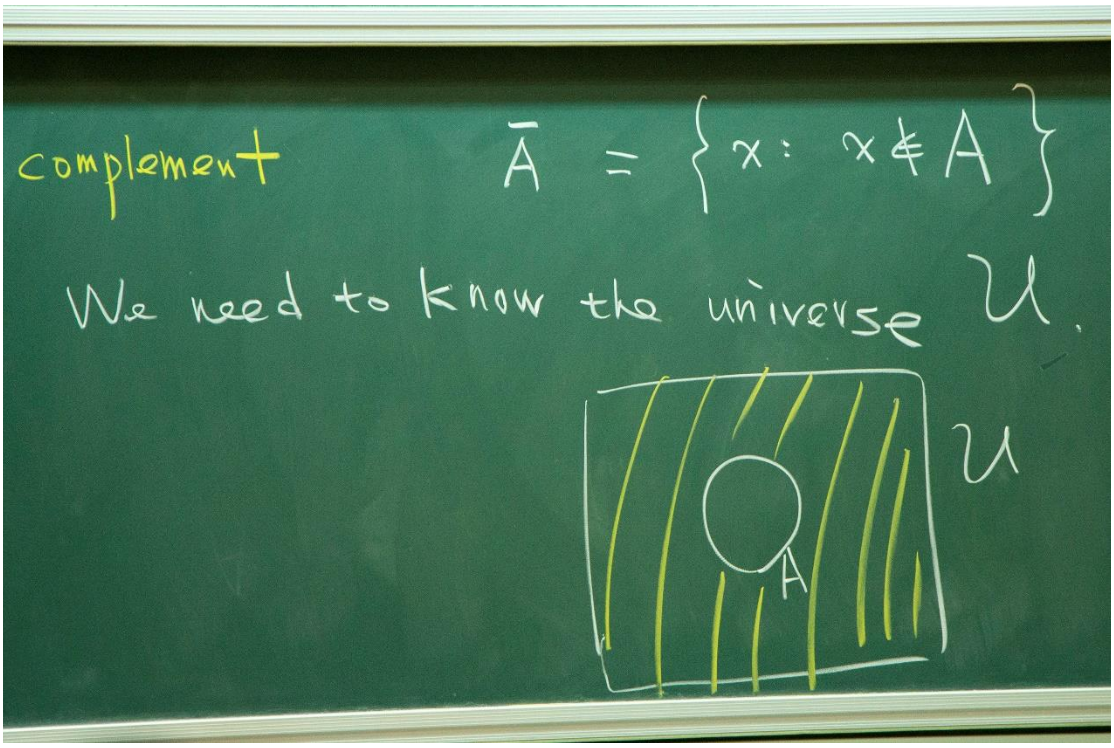
intersection $A \cap B = \{x : x \in A \text{ and } x \in B\}$

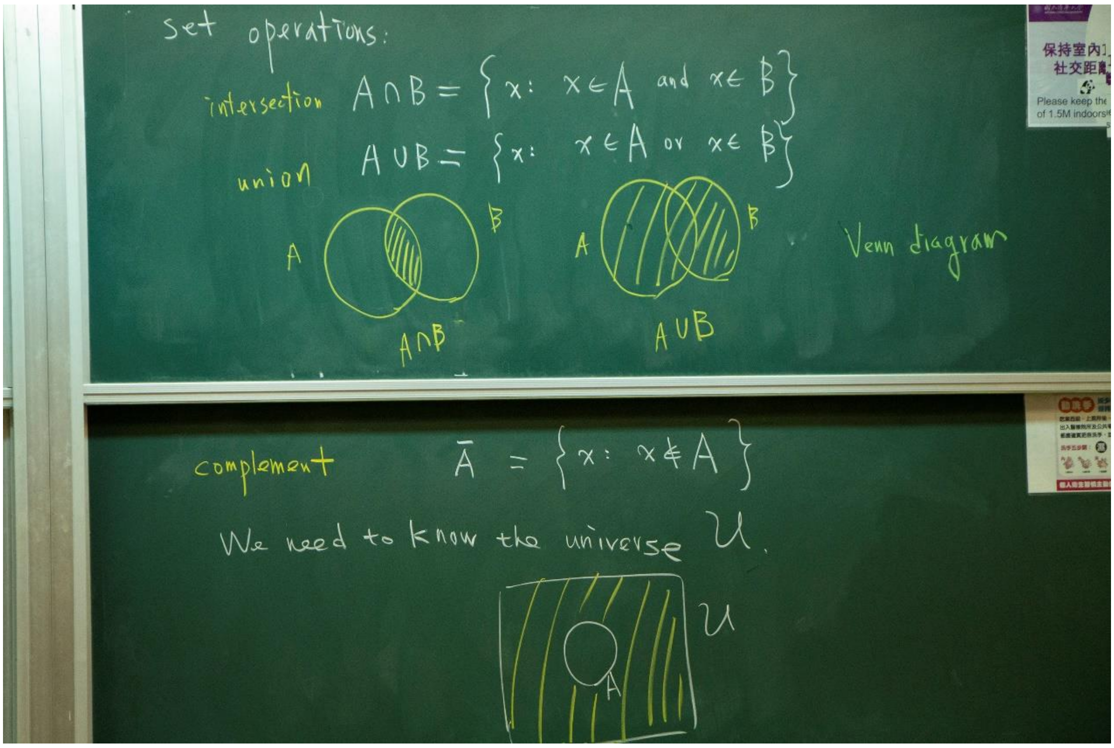
union $A \cup B = \{x : x \in A \text{ or } x \in B\}$

$A \cap B$



$A \cup B$


complement
$\bar{A} = \{x : x \notin A\}$
We need to know the universe $\mathcal{U}$.
$\mathcal{U}$
A

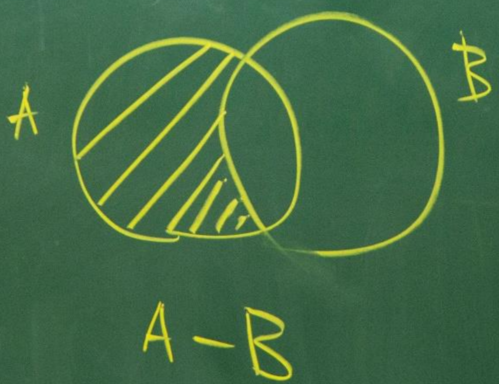

set operations:
intersection $A \cap B = \{x : x \in A \text{ and } x \in B\}$
union $A \cup B = \{x : x \in A \text{ or } x \in B\}$
A
B
$A \cap B$
A
B
$A \cup B$
Venn diagram
保持室內社交距離
Please keep the
of 1.5M indoors
complement
$\bar{A} = \{x : x \notin A\}$
We need to know the universe $\mathcal{U}$.
$\mathcal{U}$
A

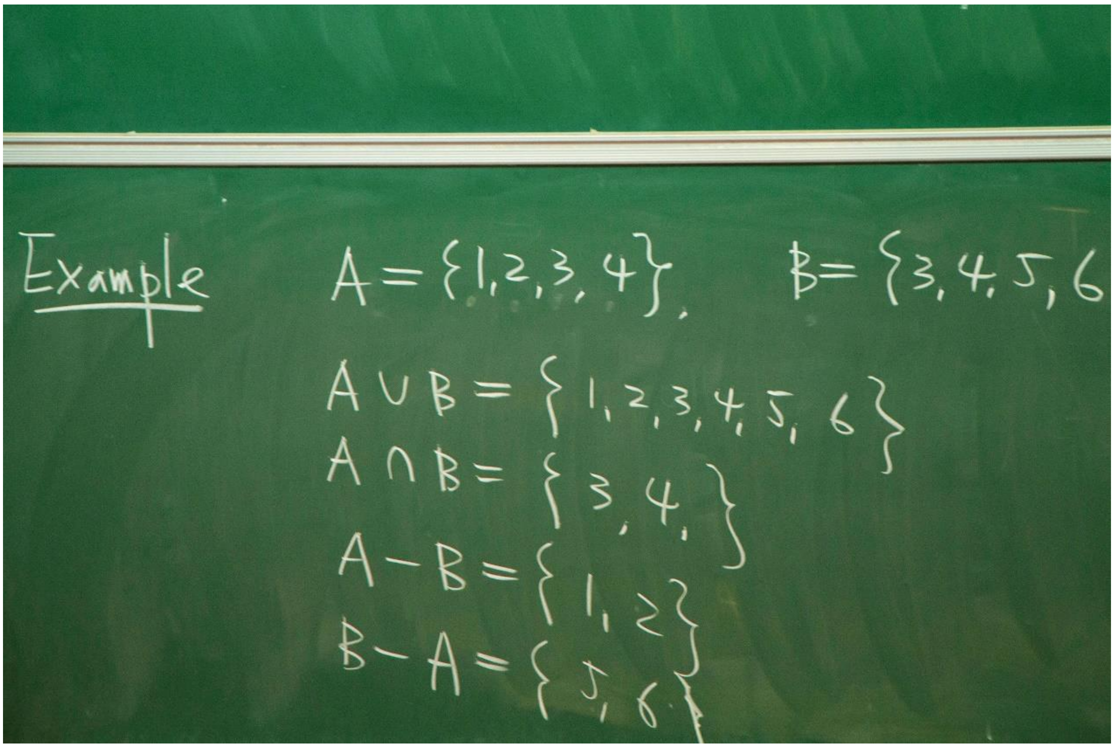
difference $A - B \triangleq A \cap \overline{B}$

(relative complement of B in A) $(A \setminus B)$

$B \triangleq A \cap \overline{B}$

B)




Example
A = {1, 2, 3, 4},
B = {3, 4, 5, 6
A ∪ B = {1, 2, 3, 4, 5, 6}
A ∩ B = {3, 4, }
A − B = {1, 2}
B − A = {5, 6, }